# Fuzzy Rule Base Interpolation Based on Semantic Revision

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## ABSTRACT

Sometimes it is no possible to have a full dense rule base as there are gaps in the information. Further, it is often necessary to deal with sparse rule base to reduce the size and the inference / control time. In such sparse rule bases the classic algorithms like the CRI of Zadeh and the Mamdani-method do not function for observation hitting into the gaps between rules. A linear fuzzy rule interpolation technique (KH-interpolation) has been introduced, that is suitable for dealing with sparse bases, however, this method often results into conclusions which are not directly interpretable.

In this paper an interpolation technique is proposed that is based on the interpolation of the semantic and interrelation of rules. This method garantiees the direct interpretability of the conclusion. The comparison of two (KH and BK) and the new interpolation method will also be discussed.

## 1. INTRODUCTION

Zadeh's crucial paper, published in 1973 [1] containing the ideas of modeling and controlling very complex (possibly nonlinear) systems through linguistically formulated rules, introduces a combination of the theory of fuzzy sets proposed in 1965 by Zadeh, and of symbolic expert control. The method in original form involved very high computational complexity. A simplified version was developed and applied soon by Mamdani [2]. Even this simplified approach has exponential space and time complexity in the terms of the number of state variables. If a fuzzy model contains $k$ variables and maximum $T$ linguistic (or other fuzzy) terms in each dimension, the order of the number of necessary rules is $O(T^k)$. This expression can be decreased by decreasing either $T$ or $k$ or both. The first method leads to sparse rule base (where $\exists i$ $\bigcup_j \mathrm{supp}\{A_{i,j}\} \subset X_i$ and $A_{i,j}$ is $j$-th fuzzy set defined on input universe $X_i$ and $i$=1..$n$; $n$ is the number of inputs). In the case of such sparse rule bases usually no rule fires in the sense of the Mamdani-, Larsen- and Takagi-Sugeno-controllers, etc. as the intersection of the observation $A^*_i$ and $A_{i,j}$ defined on $X_i$ may be empty set. The use of sparse rule bases allows us reducing the number of rules, hence reducing the problem of space and the time complexity [5]. Further, sparse rule bases can be due to incomplete knowledge or perhaps are due to automatic rule tuning [3] and has important role in hierarchically structured rule bases [4].

KH fuzzy rule interpolation interpolates among the given rules to generate a conclusion even in case when $A^*_i \cap A_{i,j} = \varnothing$ [5,6]. KH interpolation can be introduced as an interpolation technique for "fuzzy points" and the fuzziness of the interpolation is considered at each $\alpha$-level, if the semantic of fuzzy rules $R$: *If* $A_j$ *then* $B_j$ is understood as "fuzzy points" of a hypothetical mapping. Interpolation is interpretable if the existence of full orderings and metrics in each component dimension $X_i$ can be assumed, hence the definition of *partial ordering* (PO) among fuzzy sets and the "*fuzzy distance*" of two comparable sets can be introduced fully in accordance with the concept of gradual rules [7]. KH interpolation was based on the *Fundamental Equation of Rule Interpolation*:
$d(A^*, A_1):d(A^*, A_2) = d(B^*, B_1):d(B^*, B_2)$ where
$R_1$: *If* $A_1$ *then* $B_1$ and $R_2$: *If* $A_2$ *then* $B_2$ are the selected rules and fuzzy sets $A_1$ and $A_2$ are defined on input universe $X$ and $B_1$ and $B_2$ are defined on output universe $Y$. $A^*$ is the observation, and $B^*$ is the conclusion searched. $d$ denotes some kind of distance or degree of dissimilarity. In the first interpolation algorithms this distance was introduced as the set of lower and upper $\alpha$-cut distance [8], describing the relative position of two comparable convex and normal (CNF) sets unambiguously. Considering the KH-interpolation at $\alpha = 1$, and suppose that the core of the used CNF sets have one element, the classical piece wise linear interpolation is obtained fitted to the points defined by the core points of the sets. This family of interpolation techniques has various advantageous properties, however, it is applicable only for CNF-sets, and they often result into an abnormal membership function for $B^*$ that needs further transformation for obtaining a regular fuzzy set. Shi

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and Mizumoto [9], Kawase and Chen [10], Kóczy and Kovács of the KH -interpolation is always directly interpretable. The application of these conditions however leads to the restriction of the shape of the rules and of the observation that might be an obstacle of practical applications in some contexts. A method has been developed soon that reduces the problem of abnormal conclusion [12]. Further, KH-interpolation does not conserve the piece wise linearity of the rules and the observation. If e.g. an interpolation method conserves piece wise linearity it can then be simplified for widely spread set types, namely trapezoidal, triangular and crisp sets, not considering all points of the sets, but only their characteristic points, resulting in significant computational time reduction. An algorithm based on the conservation of relative fuzziness has been introduced that always guarantees the direct interpretability of the conclusion, however this approach is not applicable for certain crisp special cases [13]. A modified version has been published [14] (KHG-interpolation) that is fully applicable for CNF sets. BK-interpolation [15,16], that is a general fuzzy rule interpolation method, is applicable even for non-convex sets and always results in normal membership function, further, it conserves the piece - wise linearity if triangular or crisp sets are applied, hence it can significantly be simplified for practical application. Its main concept is different from the former ones, but it is based on the fundamental equation as well. In the BK method first a relation $\mathbf{R}_{A^i,B^i}$ is interpolated from $\mathbf{R}_{A_1,B_1}$ and $\mathbf{R}_{A_2,B_2}$ (the interpolation is done by the projected sets $A_1$, $A_2$, $B_1$ and $B_2$ applying a "solid cutting" set interpolation, that results in $A^i$ and $B^i$, in that way that $A^i$ is as close to $A^*$ as possible). In the second step $B^*$ is inferred by the specialized fixed point law linear Revision Principle (RP) method based on the similarity of $A^i$ and $A^*$. BK method does not use directly $\mathbf{R}_{A,B}$, but it functions by the projected sets.

The method introduced in this paper follows the key idea of BK method, but it interpolates the semantic relation $\mathbf{SR}_{A^*,B^*}$ and the interrelation $\mathbf{IR}_{A^*,B^*}$ instead of $\mathbf{R}_{A^i,B^i}$. Then $B^*$ is directly inferred by $\mathbf{SR}_{A^*,B^*}$ and $\mathbf{IR}_{A^*,B^*}$ from $A^*$ applying new semantic revision RP methods (SRM) that are the multi variables extension of the algorithms published by Zuliang Shen, Lia Ding and Masao Mukaidono [17]. The proposed method always guarantees the direct interpretability of the conclusion and conserves the piece - wise linearity if triangular sets are applied. The new method functions with special CNF (SCNF) sets, which core have one element, as the semantic revising RP method is applicable only for this kind of set, that is a widely spread set type in the practical applications. BK-interpolation can be simplified for SCNF sets, however it does not garantee the SCNF conclusion, hence the simplified BK method is not applicable with SCNF sets in hierarchically structured fuzzy systems. The new method garantees SCNF conclusion. The simplified version of the new method for triangular sets will also be discussed.

[11] have determined various conditions when the conclusion

## 2. DEFINITIONS

This section defines the basic concepts utilized in the interpolation for later references.

1) $s_l\{A\} = \inf(\text{support}\{A\})$ and $s_u\{A\} = \sup(\text{support}\{A\})$ where $A$ is a fuzzy set.

2) $\text{cp}\{A\}$ is the center point of fuzzy set $A$:

$$\text{cp}\{A\} = \frac{\inf(A_\alpha) + \sup(A_\alpha)}{2} \text{ and } \alpha = \text{height}\{A\}$$

where $A_\alpha$ is the $\alpha$-cut of fuzzy set $A$.. (This is the generalization of the concept of the center of the core).

3) Let $x_2 = f(x_1,p_1,p_2,c_{1m},c_{1M},c_{2m},c_{2M})$ be that function (where $x_1,p_1,p_2,c_{1m},c_{1M},c_{2m},c_{2M}$ are real numbers and $x_1,p_1 \in [c_{1m},c_{1M}]$, $x_2,p_2 \in [c_{2m},c_{2M}]$ and $c_{km} \le c_{kM}$ where $k$=1,2) which results in $x_2$. Let $x'_k = x_k - c_{km}$, $p'_k = p_k - c_{km}$, and $c'_k = c_{kM} - c_{km}$.

$$x_2 = x'_2 + c_{2m} \text{ and } x'_2 = p'_2 - (p'_1 - x'_1)\frac{bc'_2 - p'_2}{bc'_1 - p'_1}$$

$$\text{where: } b = \frac{1}{2}(1 - \text{sgn}(p'_1 - x'_1)) \text{ and } \text{sgn}(a) = \begin{cases} 1 & a > 0 \\ 0 & a = 0 \\ -1 & a < 0 \end{cases}$$

4) Let $x_2 = f^{\mp}(x_1,p_1,p_2,c_{1m},c_{1M},c_{2m},c_{2M})$ be that function (where $x_1,p_1,p_2,c_{1m},c_{1M},c_{2m},c_{2M}$ are real numbers and $x_k,p_k \in [c_{km},c_{kM}]$ and $c_{km} \le c_{kM}$ where $k$=1,2) which results in $x_2$. Let $x'_k = x_k - c_{km}$, $p'_k = p_k - c_{km}$, and $c'_k = c_{kM} - c_{km}$.

$$x_2 = x'_2 + c_{2m} \text{ and } x'_2 = p'_2 \pm (p'_1 - x'_1)\frac{\pm bp'_2 - \frac{1}{2}(1 \pm b)c'_2}{-bp'_1 - \frac{1}{2}(1-b)c'_1}$$

$$\text{where } b = \text{sgn}(p'_1 - x'_1) \text{ and } \text{sgn(a)} = \begin{cases} 1 & a > 0 \\ 0 & a = 0 \\ -1 & a < 0 \end{cases}$$

(+) is used for positive and (-) is for negative interrelation.

5) $\psi$ is the interrelation operator.

$$\psi = \begin{cases} +1 & \text{for positive interrelation} \\ -1 & \text{for negative interrelation} \end{cases}$$

## 3. MULTI VARIABLES SEMANTIC REVISING METHODS

In this section a multi variables extension of SRM-I/II method is proposed. In order to infer a conclusion two semantics factor have to be considered. Suppose that a rule *If A then B* is given where $A$ and $B$ are defined on $X = [x_m, x_M]$ and $Y = [y_m, y_M]$, respectively. Conclusion $B^*$ defined on $Y$ is inferred to observation $A^*$ defined on $X$. The first semantics factor is the direction of change [17]. For instance, assuming the rule 'if $A$ is small then $B$ is large' and the fact '$A^*$ is very small' are given, there can be some different semantics viewpoints for deducing $B^*$. The one is by the understanding that 'the smaller $A$ is the larger $B$ is'. The other is that 'the smaller $A$ is the smaller $B$ is'. The former is called *inverse inference* (-), the latter is called *compliance inference* (+). The second semantics factor is the corresponding relationship between $A$ and $B$. As shown in figure 1 the corresponding part for CD in $A$ must be either cd or df in $B$. For instance, there are two points $y_1$ and $y_2$ on $Y$, which can correspond the point $x$ on $X$ . We have to decide which one shall be used by the semantics of the corresponding relationship. Consequently, there are two corresponding relationships (CD$\rightarrow$cd, $x \rightarrow y_1$) and (CD$\rightarrow$df, $x \rightarrow y_2$). The former is called *positive inference* ($\uparrow\uparrow$), and the latter is called *negative inference* ($\uparrow\downarrow$).

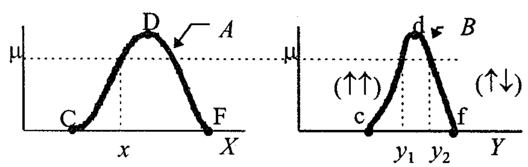


Figure 1

Suppose that rule $R$: if $A_1$ and .. and $A_n$ then $B$ is given, where $n$ is the number of inputs and $A_i$ is defined on $X_i$. and $B$ is defined on $Y$. $B^*$ is searched to $A^*_i$. For the interpolation technique we slightly modify the SRM-I and II methods and based on this modification we define an MSRM-I and II ( multi variables SRM-I and II), where cp$\{A^*_i\}$=cp$\{A_i\}$ and support$\{A^*_i\}$=support$\{A_i\}$.

### 3.1 MSRM-I .

**Characterization of the interrelation**

$$\mathbf{IR}^{(\mathrm{I})}{}_{A_i,B} = \{(x_i, y) | x_i \in \mathrm{supp}\{A_i\}, y = I_i(x_i) =$$

$$= f^{\mp}(x_i, \mathrm{cp}\{A_i\}, \mathrm{cp}\{B\}, s_l\{A_i\}, s_u\{A_i\}, s_l\{B\}, s_u\{B\})\}$$

(-) is used for negative and (+) is used for positive interrelation that is separately defined for each input dimension. $f^{\mp}$ is a piece vise linear function, so the interrelation can be defined by the characteristic points as:

$$(x_{c,i}, y_c), (x_{l,i}, y_l), (x_{u,i}, y_u) \in \mathbf{IR}^{(\mathrm{I})}{}_{A_i,B}\,, \qquad (1)$$

where

$x_{c,i} = \mathrm{cp}\{A_i\}$ ; $y_c = \mathrm{cp}\{B\}$ ; $x_{l,i} = s_l\{A_i\}$ ; $x_{u,i} = s_u\{A_i\}$ ;

$y_l = s_l\{\mathrm{B}\}$ , $y_u = s_u\{\mathrm{B}\}$

**Characterization of the semantic relation**

Then the semantic relation is defined on $[0,1]^2$ (the space of the membership degrees of $A_i$ and $B$):

$$\mathbf{SR}_a^{(\mathrm{I})}{}_{A_i,B} = \{(s,t) | s = \mu_{A_i}(x), t = \mu_B(y),$$

$$x_i \in [x_{l,i}, x_{c,i}], (x_i, y) \in \mathbf{IR}^{(\mathrm{I})}{}_{A_i,B}\}$$

$$\mathbf{SR}_b^{(\mathrm{I})}{}_{A_i,B} = \{(s,t) | s = \mu_{A_i}(x), t = \mu_B(y),$$

$$x_i \in [x_{c,i}, x_{u,i}], (x_i, y) \in \mathbf{IR}^{(\mathrm{I})}{}_{A_i,B}\}$$

$$\mathbf{SR}^{(\mathrm{I})}{}_{A_i,B} = \mathbf{SR}_a^{(\mathrm{I})}{}_{A_i,B} + \mathbf{SR}_b^{(\mathrm{I})}{}_{A_i,B}$$

**Inference by MSRM-I :**

I. Let $\mathbf{SR}^{(\mathrm{I})}{}_{A^*_i,B^*} \equiv \mathbf{SR}^{(\mathrm{I})}{}_{A_i,B}$ , $\mathbf{IR}^{(\mathrm{I})}{}_{A^*_i,B^*} \equiv \mathbf{IR}^{(\mathrm{I})}{}_{A_i,B}$

II. For any $y \in \mathrm{supp}\{B\}$ $C_i(y)$ is calculated ($i$=1..$n$) as:

If $x_i \in [x_{l,i}, x_{c,i}]$, where $(x_i, y) \in \mathbf{IR}^{(\mathrm{I})}{}_{A_i,B}$ then

$$(\mu_{A^*_i}(x_i), C_i(y)) = (\mu_{A_i}(x'_i), \mu_B(y')) \in \mathbf{SR}_a^{(\mathrm{I})}{}_{A^*_i,B^*},$$

If $x_i \in [x_{c,i}, x_{u,i}]$, where $(x_i, y) \in \mathbf{IR}^{(\mathrm{I})}{}_{A_i,B}$ then

$$(\mu_{A^*_i}(x_i), C_i(y)) = (\mu_{A_i}(x'_i), \mu_B(y')) \in \mathbf{SR}_b^{(\mathrm{I})}{}_{A^*_i,B^*},$$

$$\text{III } \mu_{B^*}(y) = \frac{\sum_i C_i(y)}{n}$$

### 3.2 MSRM-II

**Characterization of the semantic relation**

$$\mathbf{SR}^{(\mathrm{II})}{}_{A_i,B} = \{(s_i, t) | s_i = S_2(t) = t = [0,1]\}$$

**Characterization of the interrelation**

$$\mathbf{IR}^{(\mathrm{II})}{}_{A_i,B} = \{(x_i, y) | \mu_{A_i}(x_i) = s_i, \mu_B(y) = t, (s_i, t) \in \mathbf{SR}^{(\mathrm{II})}{}_{A_i,B},$$

$$(\frac{ds_i}{dx_i} \cdot \frac{dt}{dy} \psi > 0 \text{ or } \frac{ds_i}{dx_i} = \frac{dt}{dy} = 0)\}$$

The interrelation is not always piece vise linear, but the same characteristic points can be defined as in (1).

**Inference by MSRM-II :**

I. Let $\mathbf{SR}^{(\mathrm{II})}{}_{A^*_i,B^*} \equiv \mathbf{SR}^{(\mathrm{II})}{}_{A_i,B}$ , $\mathbf{IR}^{(\mathrm{II})}{}_{A^*_i,B^*} \equiv \mathbf{IR}^{(\mathrm{II})}{}_{A_i,B}$

II. $\forall t$, $t \in [0, \mathrm{height}(B)]$ $C_{i,k}(t)$ $k = 1,2$ is calculated as:

$(x_i, C_{i,k}(t)) \in \mathbf{IR}^{(\mathrm{II})}{}_{A^*,B^*}$ , where $(\mu_{A^*_i}(x_i), t) \in \mathbf{SR}^{(\mathrm{II})}{}_{A^*_i,B^*}$

$$\text{III } \mu_{B^*}\left(\frac{\sum_i C_{k,i}(t)}{n}\right) = t$$

## 4. INTERPOLATION OF THE INTER-- AND SEMANTIC RELATIONS OF TWO FUZZY RULES

This section introduces a method to interpolate the inter- and semantic relations of rules as: $R_k$: if $A_{1,k}$ and .. and $A_{n,k}$ then $B_k$ ($k$=1,2) where fuzzy sets $A_{i,k}$ are defined on input universe

$X_i = [x_{m,i}, x_{M,i}]$ and $B_k$ are defined on output universe $Y = [y_m, y_M]$. Two interpolation algorithms, that depend on interpolation variables $\lambda_i, \lambda_b \in [0,1]$, are proposed. They result in interpolated $\mathbf{IR}_{A_i,B}$ and $\mathbf{SR}_{A_i,B}$ "between" $\mathbf{IR}_{A_{i,k},B_k}$ and $\mathbf{SR}_{A_k,B_k}$ based on MSRM-I/II.

### 4.1 Interpolation for MSRM-I.

$\mathbf{IR}_{A_{i,k},B_k}$ and $\mathbf{SR}_{A_{i,k},B_k}$ are characterized as described in section 3.1. Interpolation is separately done for each input universe.

**Interpolated interrelation**

$$\mathbf{IR}^{(\mathrm{I})}{}_{A_i,B} = \{(x_i, y) | x_i = (1-\lambda_i)x_{i,1} + \lambda_i x_{i,2},$$
$$y = (1-\lambda_b)y_1 + \lambda_b y_2,$$
$$x_{i,2} = f(x_{i,1}, \mathrm{cp}\{A_{i,1}\}, \mathrm{cp}\{A_{i,2}\}, s_l\{A_{i,1}\}, s_u\{A_{i,1}\},$$
$$s_l\{A_{i,2}\}, s_u\{A_{i,2}\}), (x_{i,k}, y_k) \in \mathbf{IR}^{(\mathrm{I})}{}_{A_{i,k},B_k}\}$$

The interrelation functions are piece - wise linear, which implies that it is enough to calculate only by the characteristic point as

$$(x_{c,i}, y_c), (x_{l,i}, y_l), (x_{u,i}, y_u) \in \mathbf{IR}^{(\mathrm{I})}{}_{A_i,B}, \tag{2}$$

where

$$x_{c,i} = (1-\lambda_i)\mathrm{cp}\{A_{i,1}\} + \lambda_i \mathrm{cp}\{A_{i,2}\},$$
$$y_c = (1-\lambda_b)\mathrm{cp}\{B_1\} + \lambda_b \mathrm{cp}\{B_2\},$$
$$x_{l,i} = (1-\lambda_i)s_l\{A_{i,1}\} + \lambda_i s_l\{A_{i,2}\},$$
$$x_{u,i} = (1-\lambda_i)s_u\{A_{i,1}\} + \lambda_i s_u\{A_{i,2}\},$$
$$y_l = (1-\lambda_b)y_{l,1} + \lambda_b y_{l,2};\ y_u = (1-\lambda_b)y_{u,1} + \lambda_b y_{u,2}$$
$$(s_l\{A_{i,k}\}, y_{l,k}), (s_u\{A_{i,k}\}, y_{u,k}) \in \mathbf{IR}^{(\mathrm{I})}{}_{A_{i,k},B_k}$$

**Interpolated semantic relation**

$$\mathbf{SR}^{(\mathrm{I})}{}_{A_i,B} = \{(s,t) | s = (1-\lambda_i)s_1 + \lambda_i s_2, t = (1-\lambda_b)t_1 + \lambda_b t_2,$$
$$s_k = \mu_{A_{i,k}}(x_{i,k}), t_k = \mu_B(y_k), x_{i,2} = f(x_{i,1}, \mathrm{cp}\{A_{i,1}\}, \mathrm{cp}\{A_{i,2}\}, s_l\{A_{i,1}\},$$
$$s_u\{A_{i,1}\}, s_l\{A_{i,2}\}, s_u\{A_{i,2}\}), (x_{i,k}, y_k) \in \mathbf{IR}^{(\mathrm{I})}{}_{A_{i,k},B_k}\}$$

### 4.2 Interpolation for MSRM-II

$\mathbf{SR}^{(\mathrm{II})}{}_{A_{i,k},B_k}$ is characterized as in section 3.2.

The interrelation, including $\mathbf{SR}^{(\mathrm{II})}{}_{A_{i,k},B_k}$, can be defined on $X \times Y \times S$ in a specialized form for the interpolation as:

$$\mathbf{IR}_{\mathrm{S}}^{(\mathrm{II})}{}_{A_{i,k},B_k} = \{(x_i, y, \alpha) | \mu_{A_{i,k}}(x_i) = \alpha, \mu_{B_k}(y) = \alpha, \alpha \in S = [0,1]\}$$
$$\left(\frac{d\alpha}{dx_i} \cdot \frac{d\alpha}{dy}\psi > 0 \text{ or } \frac{d\alpha}{dx_i} = \frac{d\alpha}{dy} = 0\right)\}$$

**Interpolated interrelation**

The interpolated interrelation function is:

$$\mathbf{IR}_{\mathrm{S}}^{(\mathrm{II})}{}_{A_i,B} = \{(x_i, y, \alpha) | x_i = (1-\lambda_i)x_{i,1} + \lambda_i x_{i,2},$$
$$y = (1-\lambda_b)y_1 + \lambda_b y_2, (x_{i,k}, y_k, \alpha) \in \mathbf{IR}^{(\mathrm{II})}{}_{A_{i,k},B_k}\}$$

$\mathbf{IR}^{(\mathrm{II})}{}_{A_i,B}$ is the orthogonal projection of points $(x_i, y, \alpha) \in \mathbf{IR}_{\mathrm{S}}^{(\mathrm{II})}{}_{A_i,B}$ to $X \times Y$ as:

$$\mathbf{IR}^{(\mathrm{II})}{}_{A_i,B} = \{(x_i, y) | (x_i, y, \alpha) \in \mathbf{IR}_{\mathrm{S}}^{(\mathrm{II})}{}_{A_i,B}, \alpha \in S = [0,1]\}; \tag{3}$$

equ. (3) is not piece wise linear, but the same characteristic points can be defined as in (2).

## 5. FUZZY RULE INTERPOLATION

As mentioned above, applying interpolation PO must be defined among rules. In the proposed method PO is defined as in the BK-method: $F \prec G \Leftrightarrow \mathrm{rp}\{F\} \le \mathrm{rp}\{G\}$, where $F$ and $G$ are fuzzy sets and $\mathrm{rp}\{A\}$ is the reference point of fuzzy set $A$. Let us define the distance between two fuzzy sets as a crisp distance with the distance of their reference points:

$$d^c(A_1, A_2) = d(\mathrm{rp}\{A_1\}, \mathrm{rp}\{A_2\}) = |\mathrm{rp}\{A_1\} - \mathrm{rp}\{A_2\}|.$$

To avoid the problem of abnormal membership function for $B^*$ no other distance will be calculated, and all points of the membership function will be generated by this distance as a reference. Let us define the reference point of sets as $\mathrm{rp}\{A\} = \mathrm{cp}\{A\}$ (That is the same as in the BK method). From now cp is used in the equations instead of rp.

Once the "closest" neighbor rules $R_1$ and $R_2$ of $A^*$ are selected, as in the former interpolation methods, an interpolated semantic relation and interrelation can be generated to infer $B^*$ to $A^*$. For simplicity let us consider a sparse fuzzy rule base, where the selected neighbor rules of the observation $A^*_i$ are $R_k$: if $A_{1,k}$ and .. and $A_{n,k}$ then $B_k$. Let us denote the rules so that $B_1 \prec B_2$. The main steps of interpolation:

**The main steps of the new method:**

1) Selecting the neighbor rules of the observations, that are $R_1$ and $R_2$. Let us denote the rules so that $B_1 \prec B_2$.

2) Generating $\mathbf{SR}_{A_{i,k},B_k}$ and $\mathbf{IR}_{A_{i,k},B_k}$ ($k$=1,2). If $A_{i,1} \prec A_{i,2}$ then positive or else negative interrelation $\mathbf{IR}_{A_i,B}$ is used.

3) Interpolating $\mathbf{SR}_{A_i,B}$ and $\mathbf{IR}_{A_i,B}$.

In order to define a linear interpolation let

$$\lambda_i = \frac{\mathrm{cp}\{A^*_i\} - \mathrm{cp}\{A_{i,1}\}}{\mathrm{cp}\{A_{i,2}\} - \mathrm{cp}\{A_{i,1}\}};\ \lambda_b = \frac{\sum_{i=1}^{n} \mathrm{cp}\{A^*_i\} - \mathrm{cp}\{A_{i,1}\}}{\sum_{i=1}^{n} \mathrm{cp}\{A_{i,2}\} - \mathrm{cp}\{A_{i,1}\}} \tag{4}$$

4) Deducing $B^*$

It is supposed that the interpolated interrelation and the semantic relation can locally be used to infer $B^*$ in the surroundings of the "position" of $A^*_i$. In order to apply the inference algorithm MSRM-I/II described in section 3 this interrelation must be extended ($\mathbf{IR}_{eA_i,B}$) to the size of support$\{A^*_i\}$ satisfying two extreme cases as if support$\{A^*_i\}$=$[x_{m,i}, x_{M,i}]$, then let $x_{m,i}$, $y_{m,i}$, $x_{M,i}$ and $y_{M,i}$ be contained in $\mathbf{IR}_{eA_i,B}$ and if support$\{A^*_i\} = x_{c,i}$ then $\mathbf{IR}_{eA_i,B}$ contains only one point as $(x_{c,i}, y_{c,i})$.

$$\mathbf{IR}_{eA_i,B} = \{(x_i, y) | x_i = f(x'_i, x_{c,i}, x_{c,i}, x_{l,i}, x_{u,i}, s_l\{A^*_i\}, s_u\{A^*_i\}),$$
$$y = f(y', y_c, y_c, y_l, y_u, a, b), (x'_i, y') \in \mathbf{IR}_{A_i,B}\}$$

$$d_{1,i} = x_{c,i} - x_{m,i},\ d_{6,i} = x_{M,i} - x_{c,i},\ d_{2,i} = x_{c,i} - s_l\{A^*_i\},$$
$$d_{3,i} = x_{c,i} - x_{l,i},\ d_{4,i} = x_{u,i} - x_{c,i},\ d_{5,i} = s_u\{A^*_i\} - x_{c,i}$$

and $h_i(p) = \begin{cases} p & \text{for positive interrelation} \\ 7-p & \text{for negative interrelation} \end{cases}$

$$a = f_R^{+}\left( -\frac{\sum_{i=1}^{n} d_{h_i(2),i}}{n}, -\frac{\sum_{i=1}^{n} d_{h_i(3),i}}{n}, y_l, -\frac{\sum_{i=1}^{n} d_{h_i(1),i}}{n}, 0, y_m, y_c \right)$$

$$b = f_R^{+}\left( \frac{\sum_{i=1}^{n} d_{h_i(5),i}}{n}, \frac{\sum_{i=1}^{n} d_{h_i(5),i}}{n}, y_u, 0, \frac{\sum_{i=1}^{n} d_{h_i(6),i}}{n}, y_c, y_M \right)$$

Let $\mathbf{IR}_{A^*_i,B^*} \equiv \mathbf{IR}_{eA_i,B}$, $\mathbf{SR}_{A^*_i,B^*} \equiv \mathbf{SR}_{A_i,B}$

Then the multi variables semantic and interrelation based inference can be applied to generate a conclusion.

If the observation $\forall i$, $A^*_i$ is identical with $A_{i,k}$ : cp$\{A^*_i\}$ = cp$\{A_{i,k}\}$, hence $\lambda_i = 1$ or $\lambda_i = 0$ then the interpolation results in $\mathbf{SR}_{A^*_i,B^*}$ and $\mathbf{IR}_{A^*_i,B^*}$ that is identical with $\mathbf{SR}_{A_{i,k},B_k}$ and $\mathbf{IR}_{A_{i,k},B_k}$, thus the resulted $B^*$ is equal to $B_j$. Thus the proposed method holds the non-deviation property. Consequently, the proposed interpolation technique are fitted to the interpolation rules.

Applying MSRM-I and II in one variables case the interpolated center point is:

$$(\mathrm{cp}\{A^*\}, \mathrm{cp}\{B^*\}) \equiv (\mathrm{cp}\{A\}, \mathrm{cp}\{B\}) \equiv$$
$$(x_c = (1-\lambda)\mathrm{cp}\{A_1\} + \lambda \mathrm{cp}\{A_2\}, y_c = (1-\lambda)\mathrm{cp}\{B_1\} + \lambda \mathrm{cp}\{B_2\}) \in \mathbf{IR}_{A,B}$$

Consequently, if SCNF sets are used and the interpolation is considered at $\alpha = 1$ the classical piece wise linear interpolation is obtained fitted to the points defined by the core points of the sets in fully according with the KH- and BK-interpolation. In order to define a non-linear interpolation equ. (4) is replaced by a non-linear function.

## 6. COMPARISON BETWEEN THE MSRM-I AND II AND THE GENERAL INTERPOLATION METHOD (BK INTERPOLATION)

As mentioned above the key idea of the BK method can be separated into two steps. In the firs step a rule $R^i$ is interpolated, then in the second step a conclusion is generated based on the similarity of $R^i$ and the observations using an RP method. In the publications a solid cutting set interpolation algorithm and a specialized fixed point law has been proposed for these two steps to be applicable for arbitrary shaped fuzzy sets. If we use SCNF sets the solid cutting algorithm can be simplified in two ways applying the classical linear interpolation function that is for given points $x_1$ and $x_2$, depending on variables λ=[0,1] as

$$x^i = f^i(x_1, x_2, \lambda) = (1-\lambda)x_1 + \lambda x_2 :$$

**Simplified solid cutting set interpolation techniques for SCNF sets.**

$A^i = f^i{}_{\mathrm{I}}(A_1, A_2, \lambda)$ that is based on the interpolation of membership values of the corresponding elements. Suppose that fuzzy set $A_1$, $A_2$ and interpolation parameter λ=[0,1] are given. Let the interpolated set

$$\mu_{A^i}(x) = f^i(\mu_{A_1}(x_1), \mu_{A_2}(x_2), \lambda),\ \text{where}$$
$$x_1 = f(x, \mathrm{cp}\{A^i\}, \mathrm{cp}\{A_1\}, s_l\{A^i\}, s_u\{A^i\}, s_l\{A_1\}, s_u\{A_1\})$$
$$x_2 = f(x, \mathrm{cp}\{A^i\}, \mathrm{cp}\{A_2\}, s_l\{A^i\}, s_u\{A^i\}, s_l\{A_2\}, s_u\{A_2\})$$

and $x \in [s_l\{A^i\}, s_u\{A^i\}]$, $\mathrm{cp}\{A^i\} = f^i(\mathrm{cp}\{A_1\}, \mathrm{cp}\{A_2\}, \lambda)$

$$s_l\{A^i\} = f^i(s_l\{A_1\}, s_l\{A_2\}, \lambda),\ s_u\{A^i\} = f^i(s_u\{A_1\}, s_u\{A_2\}, \lambda)$$

$A^i = f^i{}_{\mathrm{II}}(A_1, A_2, \lambda)$ that is based on the interpolation of the fuzziness of each α-cut of the given sets. Let the interpolated set

$$s_l\{A^i{}_\alpha\} = f^i(s_l\{A_{1\alpha}\}, s_l\{A_{2\alpha}\}, \lambda),$$
$$s_u\{A^i{}_\alpha\} = f^i(s_u\{A_{1\alpha}\}, s_u\{A_{2\alpha}\}, \lambda)$$

where $A_\alpha$ is the α-cut of fuzzy set $A$.

If in the second step of the BK method the MSRM-I is applied and $f^i_{\mathrm{I}}$ is used for set interpolation then the MSRM-I based interpolation proposed in this paper is obtained. If MSRM-II and $f^i_{\mathrm{II}}$ are applied then MSRM-II based interpolation is obtained. It can be said consequently, that the MSRM-I interpolation considers the membership values of the corresponding elements and the MSRM-II interpolation considers the fuzziness of each α-cut of the given sets in fully accordance with main concept of the semantic revision RP methods, and the interpolation can be separately done at each α-level like the KH method. The proposed MSRM-I/II based interpolations conserve the piece-wise linearity for triangular sets, further, if triangular sets are used then the MSRM-I and II based interpolation and the BK with solid cutting and fixed point law RP method result in the same solution. The MSRM-I and II interpolations can be applied only for SCNF sets, however, they have an important advantageous, namely, they always result in SCNF set unlike the BK method with solid cutting and fixed point law RP method, that might result in a non-convex set. It has important role in hierarchically structured systems if simple SCNF sets are used.

The comparison of the KH and the BK method using triangular sets has been published in [18]. The comparison of the KH and proposed methods with triangular sets is the same as [18], because in this case the general method and the SRM-I and II based algorithms are identical as mentioned above.

## 7. CONCLUSION

In this paper two fuzzy rule base interpolation techniques are proposed that are based on the MSRM RP methods. These new techniques are theoretically different from the BK method, however the new methods are special cases of the BK method using proper solid cutting techniques and RP method as defined in this paper. This implies that the new methods maintain the important advantageous of the general method, namely, they conserve the piece wise linearity if triangular sets are applied and always result in a directly interpretable conclusion unlike the KH algorithm. The new methods are simpler than the BK method as they are applicable only for SCNF sets, however, they result always in SCNF sets, that is important in hierarchically structured systems.

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